# Labour in the RBC model 

Pablo Garcia

Fall 2023

## Setup

- Standard RBC model seen in class.
- But endogenous labour demand \& supply.


## Household

- She supplies labour, $n_{t}$, measured in hours.
- She gets utility from consumption, $c_{t}$, and leisure, $1-n_{t}$.
- She earns a wage, $w_{t}$, which she takes as given (perfect competition).
- Utility function:

$$
u\left(c_{t}, n_{t}\right)=\log c_{t}+\theta \log \left(1-n_{t}\right)
$$

- Budget constraint:

$$
c_{t}+i_{t}=r_{t} k_{t}+\underbrace{w_{t} n_{t}}_{\text {Wage income }}+\pi_{t}
$$

## Household

- New first order condition:

$$
w=\frac{u_{n}\left(c_{t}, n_{t}\right)}{u_{c}\left(c_{t}, n_{t}\right)}=\theta \frac{c_{t}}{1-n_{t}}
$$

- FOC balances the relative price of leisure and the marginal rate of substitution (MRS) between leisure and consumption.
- Reminder: the MRS is the rate at which the household would be willing to forgo one unit of leisure for more unit of consumption at the same utility level. The MRS is then the slope of the indifference curve.


## Firm

- The representative firm runs a standard Cobb-Douglas production function

$$
y_{t}=a_{t} k_{t-1}^{\alpha} n_{t}^{1-\alpha} .
$$

- New first order condition:

$$
w_{t}=(1-\alpha)\left(\frac{k_{t-1}}{n_{t}}\right)^{\alpha} .
$$

- FOC balances the cost of labour to its marginal productivity.


## How well does the model match US data?

- Surprisingly well.
- Here we only focus on standard deviations.
- But for a detailed analysis, check the lecture notes by Eric Sims from the University of Notre Dame. He has great material on graduate macroeconomics.


## How well does the model match US data?

## Standard Deviation

|  | c | n | i | y | w |
| :--- | :---: | :---: | :---: | :---: | :---: |
| US post-war data | 0.009 | 0.019 | 0.047 | 0.017 | 0.009 |
| RBC model | 0.006 | 0.008 | 0.058 | 0.017 | 0.009 |

- (Really) Nice job at matching the volatilities of output, investment, consumption and wages.
- $\sigma_{c}<\sigma_{y} \& \sigma_{i}>\sigma_{y}$
- But hours worked are not volatile enough!
- Suggested exercise: play with the calibration of the Dynare code I provided.
- For example, how does the discount rate affect the volatility of consumption? Or the depreciation rate the volatility of investment?


## Indivisible labour supply

- OK, so how can we raise the volatility of hours worked?
- Indivisible labour supply [Hansen (1985); Rogerson (1988)]
- Intuition: infinitely elastic labor supply.
- Households do not decide how many hours to work...
- But the probability of working a fixed number of hours!
- Skipping the details, indivisible labour supply translates to a linear disutility of labour

$$
u\left(c_{t}, n_{t}\right)=\log c_{t}-\phi n_{t}
$$

- The new FOC is then



## Indivisible labour supply

- I solve this new model using the same calibration as before.
- $\phi$ is set so that $n$ at steady state is as before, $n=1 / 3$.


## Standard Deviation

|  | C | N | I | Y | W |
| :--- | :---: | :---: | :---: | :---: | :---: |
| US post-war data | 0.009 | 0.019 | 0.047 | 0.017 | 0.009 |
| Standard RBC model | 0.006 | 0.008 | 0.058 | 0.017 | 0.009 |
| Indivisible labour supply | 0.007 | 0.015 | 0.079 | 0.022 | 0.007 |

## Impulse response functions to positive TFP shock



- Hours worked, and hence output, increase more on impact.
- Consumption and investment follow.

